

Review Problems Exponential Growth and Decay

1. The book and movie *Jurassic Park* are based on the idea that dinosaur DNA might be recovered from fossils and copied in laboratories. While the possibility of recreating dinosaurs is very remote, chemists have invented a process for copying genetic material. Suppose a chemist starts the DNA duplication process with a sample that holds 7 copies of a special piece of DNA. The process triples the DNA every 8 minutes.

- a. Write a NEXT-NOW equation that can be used to calculate the number of copies of the DNA after any number of 8 minute intervals.

NEXT = $3 \cdot \text{NOW}$ Starting at 7

- b. Write an equation in the form $y = a \cdot b^x$ that can be used to calculate the number of copies of DNA after x number of 8 minute intervals.

$y =$ $7(3^x)$

- c. How many copies of the DNA would the chemist have after 80 minutes?

"every 8 minutes"
SO $x = 10$

$y = 7(3^{10}) =$ 413,343 copies

- d. Estimate when the scientist will have 100,000,000 copies of the DNA using a table or graph. Explain how you found your answer.

$y = 100,000,000$ (or $1E8$)

$x = 15$

2. Radioactive materials have many important uses in the modern world, from fuel for power plants to medical x-rays. Radioactive materials also can be very dangerous - for example it can cause cancer. The radioactive chemical strontium-90 is produced in many nuclear reactions. Extreme care must be taken in transportation and disposal of this substance. It decays very slowly - if any amount is stored at the beginning of a year, 98% of that amount will still be present at the end of the year.

- a. If 225 grams of strontium-90 are released due to an accident, how much of that substance will still be around after 1 year? After 3 years?

$$\begin{array}{l} 1 \text{ yr} \rightarrow 220.5 \text{ gr} \\ 2 \text{ yr} \rightarrow 216.09 \\ 3 \text{ yr} \rightarrow 211.768 \text{ gr} \end{array}$$

- b. What is the decay rate of strontium-90?

2%
2%

NOT 98%

- c. Write a NEXT-NOW equation that can be used to calculate the amount of strontium-90 remaining after any number of years.

Next = NOW (0.98) Starting at 225

- d. Write an equation in the form $y = a \cdot b^x$ that can be used to calculate the amount of strontium-90 remaining after x number of years.

$$y = 225(0.98^x)$$

- * e. How long is the **half-life** of strontium-90? That is, how long until half of the original amount remains? Explain how you found your answer.

$$225 \div 2 = 112.5$$

on table, $y = 112.5$

when $x = 34.5 \text{ yrs}$

- * f. When will there be less than 2 grams of the original strontium-90 that remains? Explain how you found your answer.

$$y < 2 \text{ when } x = 234$$

3. Suppose that your grandparents starting a savings account for you when you were born. Your grandparents put \$5000 into a savings account that pays 7% interest annually.

a. Write an equation in the form $y = a(b^x)$ that models this situation.

$$y = 5000 (1.07^x)$$

b. Use your equation to fill in the below table.

Yrs since birth	0	1	2	7	12	16	20
Value of account	\$5,000	5,350	5724	8,028	11,261	14,761	19,348

c. If no deposits or withdrawals are made, use your equation to find how much the account will be worth when you are 28.

$$y = 5000 (1.07^{28}) = \$33,244.19$$

d. When you are 28 years old, you decide that you want to buy a house. The house costs \$250,000. You want to make a 20% down payment. If you did not make any deposits or withdrawals, will there be enough money in the account to make the down payment? Explain.

$$250,000 * 0.20 = 50,000$$

needed for
down payment

only have
\$33,244

NO

4. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers **increased** by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994? (Don't consider a fractional part of a person.)

if 1985 is $x=0$
then 1994 is $x=9$

$$y = 285 (1.75^x) \rightarrow y = 285 (1.75^9) = 43,872 \text{ people!}$$

↑
"increase"

5. Congratulations! You've discovered a new radioactive isotope. You head to the lab to start studying your new discovery (which will make you famous) and begin by placing 32 mg in your petrie dish. As you observe the isotope, you notice that it decays at a rate of 5% every hour.

a) Write two rules to represent this situation (*HINT: how much of the isotope remains every hour?*) Let x stand for the number of hours after you began observing the new isotope.

0.95%

NEXT = NOW * 0.95 starting at 32

$$y = 32(0.95^x)$$

b) How many milligrams of the isotope will be left in the petrie dish 6 hours after you started your observations?

$$y = 32(0.95^6) = 23.5 \text{ mg}$$

c) What is the isotope's half-life?

$$32 \div 2 = 16 \quad \text{so}$$

$y = 16$ in table when
 $x \approx 13.5 \text{ hrs}$

6. The following table shows the amount of plastic produced in the United States (in billions of pounds) per year since 1960.

Years after 1960	1	11	21	31
Plastic production	0.7	2.6	4.4	6.4

a. Use your calculator to find the exponential regression equation for this data. Write it below (round to three decimal places). Explain what the growth factor means *in the context of the problem*.

$$y = 0.850(1.074^x)$$

↑ Plastic production increases by 7.4% every year after 1960

- b. Use your calculator to find a **linear** regression model to fit this data. Write it below (round to three decimal places).

$$y = 0.189x + 0.501$$

- c. Make a scatterplot of the data in your calculator. Graph **both** regression equations on the scatterplot. Based on the **graph**, which model do you think is better? Explain.

Linear

- d. Use the model you think is better to predict the amount of plastic produced in the year 2006.

$$\rightarrow x = 46$$

$$y = 0.189(46) + 0.501 = 9.195 \text{ billion pounds}$$

- e. According to the model you chose, approximately how much plastic was produced in 1960? How can you tell from your equation?

$$\rightarrow x = 0$$

About 0.501 billion pounds
(this is the y-intercept!)

- f. According to the model you chose, when will the amount of plastic produced be 10 billion pounds per year?

$$10 = 0.189x + 0.501$$

$$x = 50$$

$$\begin{aligned} 10 &= 0.189x + 0.501 \\ -0.501 & \quad -0.501 \\ \hline 9.499 &= 0.189x \\ \frac{9.499}{.189} &= \frac{0.189x}{.189} \\ 50 &= x \end{aligned}$$

7. Your parents invest \$3,080 in a bank account when you start preschool, in hopes that the money can be used for your family to tour Europe before you start college. The account earns 3.9% annual interest. How much is in the bank account 15 years later, when you graduate high school?

$$y = 3080(1.039^x)$$

$$y = 3080(1.039^{15}) = \$5,467.44$$

$x = \#$ yrs after 1939

8. In 1939, the population of Geekville was 12,562. If the growth rate is 4.8%, what was the population in 1976?

Let 1939 represent $x=0$
so 1976 is $x=37$

$$y = 12562(1.048^x)$$
$$y = 12562(1.048^{37})$$
$$= 71,191 \text{ people}$$

9. Algebraically, find an equation for an exponential function that passes through (2, 5) and (3, 20).

$$y = a(b^x)$$

$$20 = a(b^3)$$

$$5 = a(b^2)$$

Divide: $\frac{20}{5} = \frac{ab^3}{ab^2}$

* Larger "b" exponent must be on top!

$$\boxed{4 = b}$$

Plug "b" into one of your equations:

$$5 = a(b^2)$$

$$5 = a(4^2)$$

$$\frac{5}{16} = \frac{a \cdot 16}{16}$$

$$\boxed{.3125 = a}$$

$$\boxed{y = .3125(4^x)}$$