Holleran Name: _____

IAG 2 Skinny

Midterm Exam

Review Packet

Answers to Some Commonly Asked Questions:

- This exam is over the *entire* semester.
- You get a 3x5 notecard, front and back, to use on the exam. This card will be provided by Ms. H and turned in with the midterm. You will not be provided with any equations on the actual test.
- Roughly 90% of the questions on the final come from old tests, quizzes, and review packets. The numbers in the problems will be changed.
- You have an hour and a half to take the exam.
- This is NOT a multiple choice exam—the format of the exam is the same as all the other tests you have taken in IAG 2 this year.
- How to study—PRACTICE, over and over and over! Start by re-reading all the notes (found under the "Note" tab in the binder), work on the Midterm Exam Review Packet you are given, identify your areas of weakness, then find the quizzes and tests that correspond to these topics. Re-do the problems on a separate sheet of paper and check your answer against the original answer. Do NOT simply "look over" a problem—this is a waste of your time and is not an effective way to study. If you do not have the correct answer written down on the original paper, see Ms. H. All review packets should be posted on the course website by Dec. 15.
- Good luck!

Unit 1 & 2 – Quadratic Functions

1. What is the general form of a quadratic function? _____

2. Write the function that gives the height of an object dropped or launched after *t* seconds: _____

3. In the general form of a quadratic function
   What does the value of "a" tell you? _____

   What does the value of "c" tell you? _____

   What does the value of "b" tell you? _____

4. Identify the "a" "b" and "c" values in the following equations.

   a.) $y = x^2 + 3x - 9$ a = _____ b = _____ c = _____

   b.) $h(t) = 12 + 24t - 16t^2$ a = _____ b = _____ c = _____

   c.) $f(x) = -18 + 3x^2$ a = _____ b = _____ c = _____

5. A pumpkin is pushed off a building 12 feet above the ground. What rule shows how the pumpkin's height $h(t)$ is related to the time elapsed since it was pushed, t? Estimate the time it takes for the pumpkin to hit the ground.

6. Imagine a two-tower suspension bridge over the Ohio River. The curve of the main suspension cable can be modeled by the function $y = 0.008x^2 - 2x + 150$, where *y* is the height of the cable above the bridge surface and *x* is the distance from one tower towards the other.
   Window: 0 < x < 250, xscl = 20; 0 < y < 150, yscl = 20.

   a. What is the height of the two towers?

   b. What is the shortest distance from the cable to the bridge surface and where does it occur?

c. When is the suspension cable at least 70 feet above the bridge surface?

7. Jasmine threw a softball from the outfield. The table below gives the time since the ball was thrown (in seconds) and the height of the ball (in feet).

Time (Sec)	0	0.5	1.0	1.5	2.0	2.5
Height (ft)	5	23.5	34	36.5	31	17.5

a. Bobby is trying to figure out a function rule that will match the data. He wants the rule to be in the form $h(t) = -16t^2 + bt + c$. Find the values of *b* and *c*. (Hint: *b* is the initial velocity!)

b. Explain what the values of *b* and *c* tell you about Jasmine's throw.

c. If nobody catches the ball, did it hit the ground before or after 3 seconds passed? Explain.

d. Write the height equation using the values of *b* and *c* you found in part (a). Then graph the equation and find the maximum height and the time it occurs.
Window: 0<x<4, xscl = 0.25, -4<y<40, yscl = 2

8. Describe the relationship between the graphs of the following function pairs. Do not use your calculator. See the following example:

Example: $y = x^2$ and $y = x^2 - 8x$ *"The second equation's graph is moved to the right of the other"*

a) $y = 2x^2$ and $y = -2x^2$

b) $y = 2x^2$ and $y = 0.5x^2$

c) $y = x^2$ and $y = x^2 + 4$

d) $y = x^2$ and $y = x^2 - 8x$

e) $y = x^2 - 3x$ and $y = x^2 - 3x + 4$

f) $y = x^2 + 2$ and $y = x^2 + 5x + 2$

g) $y = -x^2$ and $y = -x^2 + 2x$

h) $y = -3x^2$ and $y = -3x^2 - 8x$

9. A young baseball player swung his bat 3 feet above the ground and hit a baseball. After 1.2 seconds the ball has is 15.96 feet above the ground. Find the initial upward velocity of the baseball.

10. Larry and Sherry want to open a water park, and want to charge customers to use their "Water Slide of Death." Sherry asked a market research company to survey how the number of customers would depend on the admission price. Here is what the survey said:
    - The data suggested that daily number of customers *n* would be related to admission price *x* by *n* = 500 -10*x*.
    - The cost of operating the water slide attraction is $450 a day.
    - The insurance cost is $4 per customer (after all, it IS called the Water Slide of Death…)

    a.) Fill in the table below.

	Ticket Price (in $)					
	0	**9**	**18**	**27**	**36**	**49**
Number of Tickets Sold	500					
Income (in $)	0					
Operating Cost	450					
Insurance Cost	2000					
Profit ($)	-2,450					

11. Rewrite each of the following expressions in equivalent form after factoring:

a.) $8x^2 + 5x$

b.) $x^2 - 10x$

c.) $x - 4x^2$

12. Rewrite each expression in equivalent standard form ax^2+bx+c.

a.) $(x+3)(x+5)$

b.) $(x-4)(x+7)$

c.) $(x-5)(x+5)$

d.) $(x+8)^2$

e.) $6x(9-3x)$

f.) $4(2x+5)+2x(x-1)$

13. Solve each equation using algebra.

a.) $x^2 = 18$

b.) $8x + 6x^2 = 0$

c.) $4x^2 - 2x = 0$

d.) $5x^2 - 30 = 95$

e.) $9x - 2x^2 = -5$

f.) $20 + 8x^2 = 5$

14. Consider the equation $y = x^2 - 8x$

a.) What are the x-intercepts of the graph of this function?

b.) Find the coordinates of the minimum point using the x-intercepts and what you know about the symmetry of parabolas. Do NOT use the graphing part of your calculator.

15. Match each equation to the appropriate graph using information from the "a" "b" and "c" values. Each graph has the same x- and y-scale.

Rule I $y = x^2 + 2$	**Rule II** $y = x^2 - 5x + 2$
Rule III $y = -x^2 + 2$	**Rule IV** $y = -0.5x^2 + 2$
Rule V $y = x^2 + 5x + 2$	

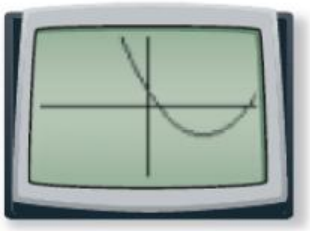
A

Rule ____

B

Rule _____

C



Rule _____

D

Rule _____

E

Rule _____

16. Write a rule for a quadratic function with a parabolic graph that has x-intercepts (2, 0) and (-6, 0) and maximum point (-2, 4). Leave in the form $f(x) = a(x - m)(x - n)$.

17. Write each product in equivalent $ax^2 + bx + c$ form.

a.) (x + 3)(x – 5)

b.) (3x + 5)(x + 2)

c.) $(x - 5)^2$

d.) (x + 10)(x – 10)

e.) $(x-7)(x+8)$

f.) $(x-9)(x-9)$

g.) $(2x+7)^2$

h.) $(5x-3)(5x+3)$

18. Find an equivalent factored form for each quadratic expression, if possible. If not possible, state "not possible."

a.) $x^2 + 6x + 8$

b.) $x^2 - 49$

c.) $x^2 + 10x - 24$

d.) $36 - 12x + x^2$

e.) $16 - 25x^2$

f.) $2x^2 - 12x - 14$

g.) $33x^2 - 6x$

19. Solve these quadratic equations by using **SQUARE ROOTS, FACTORING**, or the **QUADRATIC FORMULA** $\left(x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right)$**.** If you cannot solve, state "no real solution". Round all decimal answers to the tenths place.

a.) $x^2 - 15x + 50 = 0$

b.) $x^2 - 9x + 20 = 20$

c.) $2x^2 - 10x = 0$

d.) $5 - x^2 = -479$

e.) $x^2 - 15x = 9 - 15x$

f.) $-x^2 + 16x - 8 = 0$

g.) $x^2 - 4x + 3 = 0$

h.) $14 + 4x^2 = 30$

i.) $x^2 - 7x + 2 = 20$

j.) $2x^2 + 25x + 33 = 0$

k.) $9x^2 = 12x$

20. Without the use of your graphing calculator, find the coordinates of the x-intercepts, y-intercept, and vertex points for the graphs of the following quadratic functions.

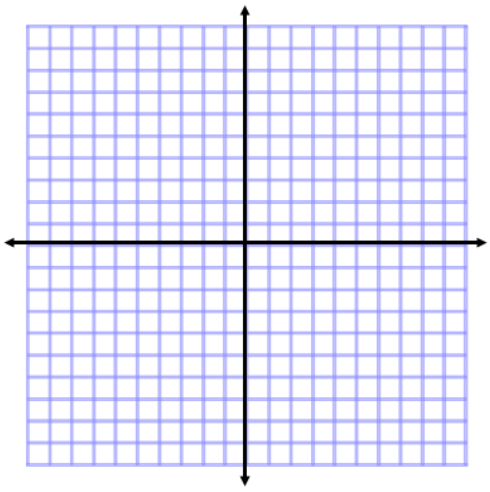
a.) $f(x) = (x - 3)(x - 8)$

b.) $f(x) = -(x - 3)(x + 5)$

21. Write a quadratic function in the form $f(x) = a(x - m)(x - n)$ that has a graph with x-intercepts at (-2, 0) and (6, 0) with y-intercept at (0, -60).

22. Sketch each of the graphs by finding, plotting and labeling the x-intercepts, y-intercept, and vertex point.

a. f(x) = $-\frac{1}{2}$ (x - 2) (x +4)



x-intercepts: (_____ , 0) (____ , 0)

y-intercept: (0, _____)

vertex: (_____ , _____)

23. Solve each of the following equations using algebra, then provide a small sketch of the graphs in the system and label all solutions on the graph.

a.) $x + 3 = \frac{10}{x}$

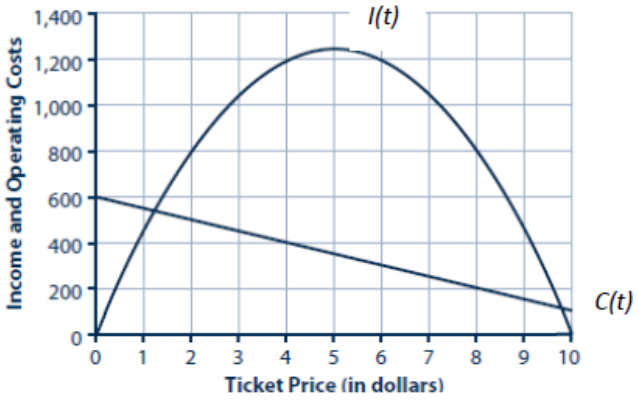
b.) $6x - 5 = x^2 - 4x + 20$

c.) $-4x = \frac{24}{x}$

24. Each year, the combined high school orchestras in Muse City stages a public concert. Based on data from previous years, the organizers decided that the income and operating costs can be represented as functions of ticket price according to the following equations and graph:

Income from ticket sales *I(t)* is related to ticket price *t* by the equation $I(t) = 500t - 50t^2$.

Cost *C(t)* of operating the concert is related to ticket price *t* by the equation $C(t) = 600 - 50t$.



a.) What ticket price(s) would generate the greatest income (approximately)? What is the greatest income?

Ticket Price(s) = _____ Greatest Income = _____

b.) For what ticket price(s) would the operating costs be equal to the income from ticket sales?

c.) For what ticket prices would the operating costs be less than the income from ticket sales?

d.) Which of the following rules gives the predicted profit *P(t)* as a function of ticket price?

I. $P(t) = -50t^2 + 550t - 600$
II. $P(t) = 50t^2 - 550t + 600$
III. $P(t) = -50t^2 - 450t + 600$

25. Solve the following system of nonlinear equations using the quadratic formula: $\begin{cases} y = 3x + 35 \\ y = 2x^2 + 14x - 15 \end{cases}$

Unit 3 -- Quadratic Functions Pt. 3

26. Factor the following:

a) $x^2 + 4x - 32$

b) $12x^2 + 5x - 2$

c) $3x^2 - 3x - 90$

d) $25x^2 - 16$

e) $-2x^2 - 6x + 56$

f) $6x^2 - 11x + 4$

g) $5x^2 - 50x + 120$

h) $9x^2 + 12x + 4$

i) $49 - 100x^2$

27. Now, using your answers from Problem (26), solve each of the equations.

a) $x^2 + 4x - 32 = 0$

b) $12x^2 + 5x - 2 = 0$

c) $3x^2 - 3x - 90 = 0$

d) $25x^2 - 16 = 0$

e) $-2x^2 - 6x + 56 = 0$

f) $6x^2 - 11x + 4 = 0$

g) $5x^2 - 50x + 120 = 0$

h) $9x^2 + 12x + 4 = 0$

i) $49 - 100x^2 = 0$

28. Express each of the following quadratic functions in (a) x-intercept form by factoring the equation and (b) vertex form by completing the square. Then find the coordinates of the given points using whichever form is easiest.

a) $f(x) = x^2 - 12x + 11$

y-intercept: (____, ____)

x-intercept form: $f(x) =$ _____

x-intercepts: (____, ____) (____, ____)

vertex form: $f(x) =$ _____

vertex point (____, ____)

b) $f(x) = x^2 + 9x + 20$

y-intercept: (____, ____)

x-intercept form: $f(x) =$ _____

x-intercepts: (____, ____) (____, ____)

vertex form: $f(x) =$ _____

vertex point (____, ____)

c) $f(x) = x^2 - 8x + 7$

y-intercept: (____, ____)

x-intercept form: $f(x) =$ _____

x-intercepts: (____, ____) (____, ____)

vertex form: $f(x) =$ _____

vertex point (____, ____)

d) $f(x) = x^2 - 2x - 80$

y-intercept: (____, ____)

x-intercept form: $f(x) =$ _____

x-intercepts: (____, ____) (____, ____)

vertex form: $f(x) =$ _____

vertex point (____, ____)

e) $f(x) = x^2 + 6x + 5$

y-intercept: (____, ____)

x-intercept form: $f(x) =$ _____

x-intercepts: (____, ____) (____, ____)

vertex form: $f(x) =$ _____

vertex point (____, ____)

29. Simplify the following complex numbers by performing the indicated operations. Leave answers in standard form, a+bi.

a) $(5 + 6i) - (3 - 9i) - (-2 - 7i)$

b) $(7 + 9i) - (2 + 11i)$

c) $(7 + 9i) + (2 + 11i)$

d) $(-2 - i)(4 + i)$

e) $i(-5 - 8i)$

f) $(1 - 2i)^2$

g) $5i + 8i \cdot i$

h) $6i - 5(10 + 4i)$

i) $-3i \cdot 6i - 3(-7 + 6i)$

j) $(3 + 3i) + (8 - 2i) - 7$

30. Use the quadratic formula to solve each of the following equations. Leave all irrational answers in exact form. Write nonreal complex numbers in standard form a+bi.

a) $2x^2 + x - 3 = 0$

b) $2x^2 + 5x - 1 = 0$

c) $5x^2 - 6x + 2 = 0$

d) $x^2 + 9x - 10 = -24$

e) $x^2 - 6x + 13 = 0$

f) $3x^2 - 18x + 30 = 0$

Unit 4 – Coordinate Geometry

31. Slopes of lines:

    a) What is the slope of a line perpendicular to a line with slope $-\frac{2}{3}$?

    b) What is the slope of a line parallel to a line with slope $-\frac{2}{3}$?

32. Given the points A(3,5) and B(2,-9), find the following.

    a) Slope =

    b) midpoint =

    c) Length =

    d) Find the equation for a line that goes through AB

33. Give the slope for:

    a) A line perpendicular to the line y = 2-5x m = _____

    b) A line parallel to y = $\frac{3}{5}x$-11 m = _____

    c) A line perpendicular to 3x + 9y = 27 m = _____

34. Give the coordinates of the image of the point (-4, 1) under each indicated transformation

Transformation	Preimage Coordinates	Image Coordinates
Translation 6 units to the right, 5 units down	(-4, 1)	
Reflection across x-axis	(-4, 1)	
Reflection across the line y = x	(-4, 1)	
270° counterclockwise rotation	(-4, 1)	
Size transformation of scale factor 5	(-4, 1)	
Clockwise rotation of 90° followed by a clockwise rotation of 270°	(-4, 1)	

35. Triangle ABC is an isosceles triangle. Is the image of triangle ABC under a 90 degrees counterclockwise rotation also an isosceles triangle? Will ABC and the image of ABC be congruent? How do you know?

36. The segment AB has coordinates of A(3, 1) & B (5, 15).

a) Find the slope.

B) What is the slope of any line perpendicular to AB?

c) Find the midpoint of segment AB.

b) Now use your answers from part (b) and (c) to find the equation of the line that would be the perpendicular bisector of segment AB.

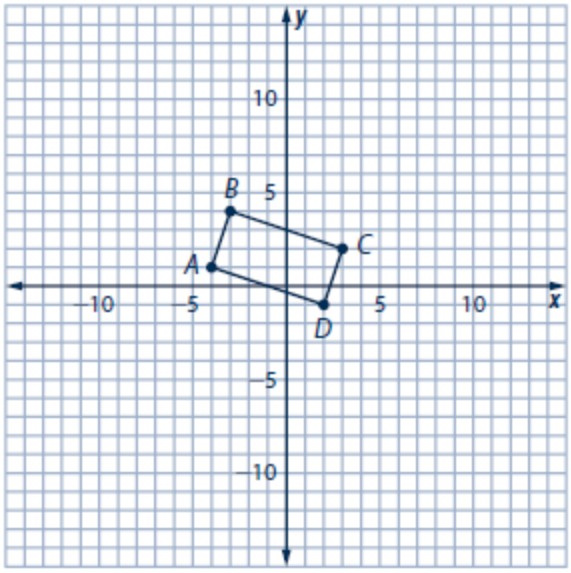
37. The area of a rectangle is $\sqrt{7}$ square inches. Find the area of the IMAGE of this rectangle after the following size transformation. Explain/Show your work.

a) After a size transformation of magnitude 5

b) After a size transformation of magnitude 0.25 (you may round answer to nearest tenth)

38. Below is a sketch of a quadrilateral with the vertices A (-4, 1), B (-3, 4), C (3, 2) and D (2, -1).

a) On the next page, transform *ABCD* by first reflecting it across the y-axis and then applying a size transformation of magnitude 3. Draw the FINAL image on the grid and label it *A''B''C''D''*. Provide coordinates of point *A''*, *B''*, *C''*, and *D''*.



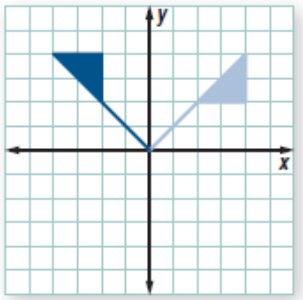
Pre-Image	Reflect over y-axis	Size Trans. of Magnitude 3
A (-4, 1)	A' _____	A'' _____
B (-3, 4)	B' _____	B'' _____
C (3, 2)	C' _____	C'' _____
D (2, -1)	D' _____	D'' _____

b) Write a rule that describes the *composite* transformation in part (a).

(x, ~~y~~) ⟶ (_____ , _____)

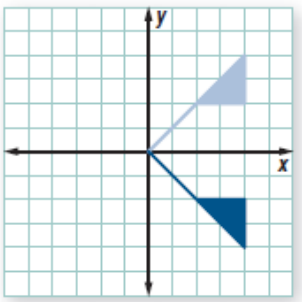
39. Preimage and image pairs of a figure under certain transformations are shown below. The image is the darker flag. In each case, identify as precisely as you can the type of transformation. Then write a coordinate rule for the transformation.

a.



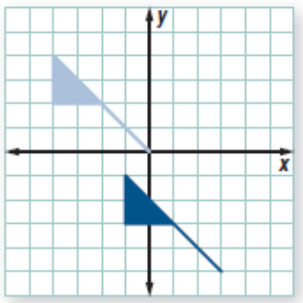
$(x, y) \rightarrow (\_\_, \_\_)$

b.



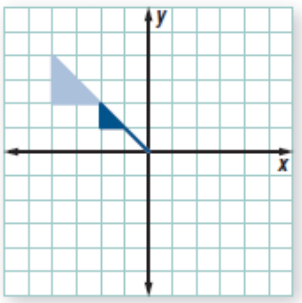
$(x, y) \rightarrow (\_\_, \_\_)$

c.



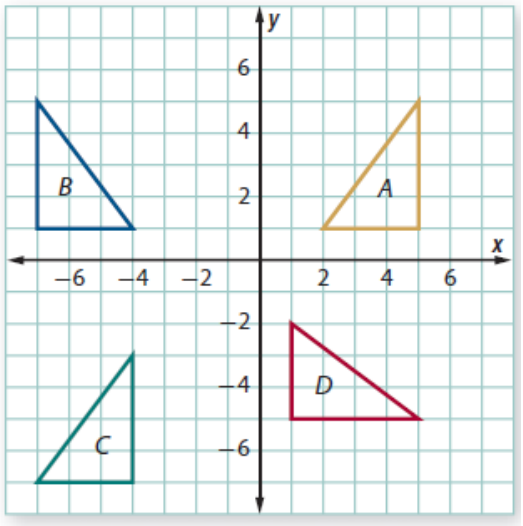
$(x, y) \rightarrow (\_\_, \_\_)$

d.



$(x, y) \rightarrow (\_\_, \_\_)$

40. Consider the coordinate grid below. Describe in words a transformation or composition of transformations that will move the first triangle onto the second.

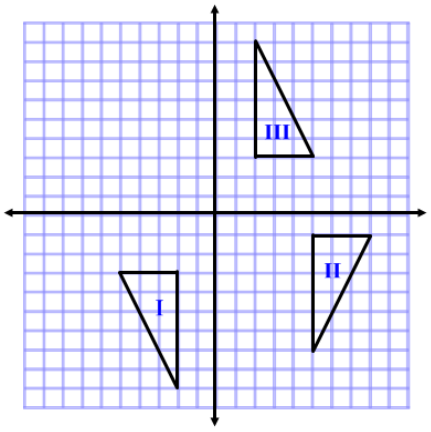


a) Transformation(s) that moves ΔA onto ΔB

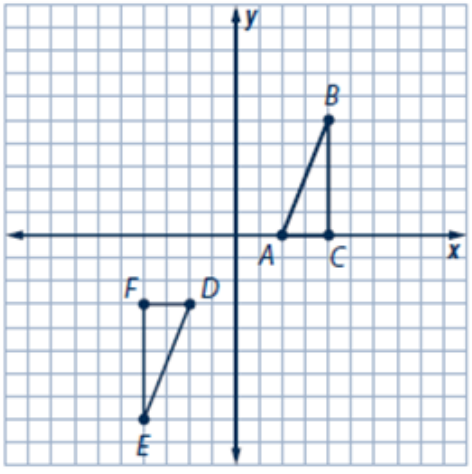
b) Transformation(s) that moves ΔA onto ΔC

c) Transformation(s) that moves ΔA onto ΔD

41. Look at triangle I and II below. Write a sentence describing the transformation, or sequence of transformations, that maps triangle I onto triangle II.



12. Refer to the coordinate grid below. The scale on each axis is one.



a. Find the length of line segment ED using the distance formula.

b. Find the length of line segment AB. What do you notice about AB and ED?

c. Which point of triangle DEF is the image of point A under your composite transformation from part b?